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Proportional Representation

Apportionment Methods
and Their Applications

Second Edition

With a Foreword by Andrew Duff MEP

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